

Definite Integral = Some Number

Indefinite Integral = family of functions  $\int f(x) dx$   
(Antiderivative)

$$\int f(x) dx = F(x) + C \quad \text{if } F'(x) = f(x)$$

**Properties of Indefinite Integrals**

$$\int k f(x) dx = k \int f(x) dx \quad \text{for any constant } k$$

$$\int (f(x) \pm g(x)) dx = \int f(x) dx \pm \int g(x) dx$$

**Power Formulas**

$$\int u^n du = \frac{u^{n+1}}{n+1} + C \quad \text{when } n \neq -1$$

$$\int u^{-1} du = \int \frac{1}{u} du = \ln |u| + C$$

(see Example 2)

**Trigonometric Formulas**

$$\int \cos u du = \sin u + C$$

$$\int \sin u du = -\cos u + C$$

$$\int \sec^2 u du = \tan u + C$$

$$\int \csc^2 u du = -\cot u + C$$

$$\int \sec u \tan u du = \sec u + C$$

$$\int \csc u \cot u du = -\csc u + C$$

**Exponential and Logarithmic Formulas**

$$\int e^u du = e^u + C$$

$$\int a^u du = \frac{a^u}{\ln a} + C$$

$$\int \ln u du = u \ln u - u + C \quad (\text{See Example 2})$$

$$\int \log_a u du = \int \frac{\ln u}{\ln a} du = \frac{u \ln u - u}{\ln a} + C$$

$$\frac{d}{dx} 3^x = 3^x \ln 3$$

$$\int 3^x = \frac{3^x}{\ln 3}$$

$$\int f(x) \underline{\underline{dx}}$$

## Exploration 1

p. 333

$$u = x^2$$

$$f(u) = u^3$$

$$f(u) \text{ as function of } x = x^6$$

$$\frac{d}{dx} x^2 + y$$

$$\frac{\partial}{\partial y} x^2 + y$$

$$\textcircled{1} \int f(u) du = \frac{u^4}{4}$$

$$\textcircled{2} \frac{u^4}{4} \rightarrow \frac{(x^2)^4}{4} = \left( \frac{x^8}{4} \right)$$

$$\textcircled{3} \int f(u) dx = \left( \frac{x^7}{7} \right)$$

$$\textcircled{4} \underline{\underline{No}}$$

$$\int \underbrace{\sin x e^{\cos x}}_{du} dx = -e^{\cos x}$$

$$u = \cos x$$

$$du = -\sin x dx$$

$$-\int e^u du = -e^u$$

$$\boxed{= -e^{\cos x} + C}$$

$$\int \underline{x \cos(2x^2)} dx$$

$$u = 2x^2$$

$$du = 4x dx$$

$$\frac{1}{4} \int \cos u du = \frac{1}{4} \sin u$$

$$= \frac{1}{4} \sin(2x^2) + C$$

$$\int \sin 3x \, dx$$

$$u = 3x \\ \frac{du}{3} = \frac{3}{3} dx \rightarrow dx = \frac{1}{3} du$$

$$\frac{1}{3} \int \sin u \, du = -\frac{1}{3} \cos u \quad \boxed{= -\frac{1}{3} \cos 3x + C}$$

Try 19-24

see examples for more scaffolding

$$\begin{array}{l}
 \int \frac{dx}{x^2+9} \rightarrow \begin{array}{l} u = \frac{x}{3} \\ du = \frac{1}{3} dx \\ dx = 3 du \end{array} \rightarrow \int \frac{3 du}{9(u)^2+9} \rightarrow \frac{3}{9} \int \frac{du}{u^2+1} \\
 \int \frac{dx}{9\left(\frac{x}{3}\right)^2+9}
 \end{array}$$

HW

sect. 6.2

# 1-16( $\frac{1}{2}$ )

19-24